Fuzzy Systems and Soft Computing
ISSN : 1819-4362

FUNDAMENTAL PROPERTIES OF OPEN BMS AND CLOSED BMS

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Abstract:
This paper aim to be introduce the binary multiset (bms) semi-open, bms pre-open,bms α-open, bms β-open, bms ω-open, bms γ-open, bms b-open and bms regular-open in bms-topological space. Additionally, we present the fundamental theorems, properties, and examples are explains in the bms-topological space approach of this new form.

Keywords:
bms semi-open, bms pre-open, bms α-open, bms ω-open, bms γ-open, bms b-open.

Mathematics Subject classification: 00A05, 03E70, 54B10, 54G20.

1 Introduction

Topology is a significant branch of mathematics dedicated to the study of spatial properties that remain invariant under continuous deformations. This field examines various types of spacesand their intrinsic characteristics, emphasizing the relationships and structures that persist even when the shapes are manipulated through stretching or bending, without tearing or gluing. The exploration of topological spaces has led to numerous advancements in mathematical theory and application. In 1991, the concept of multisets was introduced by Bilzard [4], who laid the groundwork for multiset theory and significantly advanced its development. Multisets, which allow fo multiple instances of the same element, provided a new lens through which mathematicians could analyze and understand complex structures. Building on this foundation, Nithyanathan Jothi, et.al., [7] and [10], in 2011, proposed a binary theory that addresses the continuous evolution of topological spaces, offering valuable insights into their dynamics and properties. Further contributions to the field have been made by Andrijivic[2,3], who established generalized topological spaces for semi-preopen and b-open sets. This work expanded the understanding of open and closed sets within the context of generalized topology, introducing nuanced definitions that facilitate a deeper exploration of space. Girish,et.al., [5] introduced the concept of multiset topological spaces, providing definitions, examples, theorems, and properties that enhance the theoretical framework surrounding multisets in topology. Additionally, Lellis Thivagar, et.al., [6] made noteworthy contributions by discussing the notion of -closed sets within topological spaces. Thisparticular type of closed set plays a critical role in various topological discussions and serves as an essential tool for understanding continuity and conver}ence in mathematical analysis. The term α-closed set was defined by Sundaram and Sheik John [11], further enriching the vocabulary and tools available to researchers in this area. Abd El-Monsef's [1] work focused on the exploration of ⊆-open sets and the concept of ⊆-continuous mappings, adding another layer to the understanding of open and continuous functions in the realm of topology. This exploration has implications for various applications, including the study of conver}ence and continuity in both theoretical and applied mathematics. Moreover, Priyalatha et al. [8] introduced the concept of binary multiset topological spaces, providing theorems and examples that illustrate the unique properties and applications of these structures. Their work highlights the significance of binary operations within multiset topology and opens new avenues for research and discovery. In this paper, we present novel insights into binary multiset (bms) structures, delving into various types of bms , including bms semi-open, bms semi-closed, bms pre-open, bms pre-closed, bms ⊆-open, bms α-closed, bms regular-open, bms regular-closed, bms -open, bms -closed, bms -open, bms -closed, bms b-open, bms b-closed, bms ⊆-open, and bms α-closed. Our exploration aims to enhance the theoretical framework surrounding these structures and to provide a comprehensive understanding of their implications within the broader context of topology.

2 Preliminaries and Basic Definitions

Definition 2.1. [5] An mset M drawn from the set X is represented by a function count M or C_M defined as $C_{M \to N}$ .Where N represents the set of non-negative integers. Here $C_{M(x)}$ is the number of occurrences of the element x in the mset M. We present the mset M drawn from the set X = {X_1, X_2, …,X_n} as M = {m_1/x_1, m_2, /$x_{2,}$... ,m_n/x_n}. Where mi is the number of occurrences of the element x_i = 1, 2....n in the mset M. However those elements which are not included in the mset M have zero count.

Definition 2.2. [5] A domain X, is defined as a set of elements from which mset are constructed. The mset space $[X]^w$ is the set all msets whose elements are in X such that no elements in the mset occurs more than w times. The set $[X]^{\alpha}$ is the set all msets over a domain X such that there is no limits on the number of occurrences of an elements in an msets. If X = {x_1, x_2, ...,x_k} then $[X]^w$ = {m_1/x_1, m_2/x_2, ...m_k/x_k}.

Definition 2.3. [5] Let $M \in [X]^w$ and $\tau \subseteq P^*(M)$. Then τ is called a multiset topological space of M if τ satisfies the following properties.

(i) The mset M and the empty mset φ are in τ.

(ii) The mset union of the elements of any sub collection of τ is τ.

(iii) The mset Intersection of the elements of any finite sub collection of τ is in τ.

Definition 2.4. [5] A sub mset N of M-topological space M in [X]w is said to be closed if the Mset M N is open. In discrete M-topological space every mset is an open mset as well as a closed mset. In the M-topological space PF(M)∪φ, every mset is an open mset as well as a closed mset.

Definition 2.5. [5] Given a submset A of an M-topological space M in $[X]^w$, the interior of A is defined as the mset union of all open mset contained in A and it's denoted by Int (A). i.e., Int (A) = ∪ {G ⊆ M : G is an open mset and G ⊆ A} and $C_{Int(A)(x)} = \max\{CG(x) : G \subseteq A\}$.

Definition 2.6. [5] Given a submset A of an M-topological space space M in $[X]^w$, the closure of A is defined as the mset intersection of all closed mset containing A and its denoted by Cl (A). i.e., Cl (A) = ∩ {K ⊆ M : K is a closed mset and A ⊆ K} and $C_{Cl(A)(x)} = \min\{CK(x) : A \subseteq K\}$.

Definition 2.7. [7] Let X and Y be any two no empty sets. A binary topological space X to Y is a binary structure $M \subseteq \rho(X) \times \rho(Y)$ that satisfies the following axioms.

(i) (φ, φ) and(X,Y) ∈ M.

(ii) $(A_1 \cap A_2, B_1 \cap B_2) \in M$ whenever $(A_1, B_1) \in M_1$, $(A_2, B_2) \in M_2$.

(iii) If $\{(A_\alpha, B_\alpha): \alpha \in \Delta\}$ is a family of members of M, then $(S\alpha\in \Delta A_\alpha, S\alpha\in \Delta B_\alpha) \in M$.

Definition 2.8. [10] Let (M, τ) be a multiset topological space on [X] w and N sub-mset of M. We define the following definition:

(i) A Semi-open if A ⊆ cl (int(A)) with $C_{A(x)} \leq C_{cl(int(A))(x)}$, ∀x ∈ X.

(ii) A semi-closed if int (cl(A)) ⊆ A with $C_{int(cl(A))(x)} \leq C_{A(x)}$, ∀x ∈ X.

(iii) A semi-pre-open (β − open) if A ⊆ cl (int(cl(A)) with $C_{A(x)} \leq C_{cl(int(cl(A)))(x)}$, ∀x ∈ X.

(iv) A semi-pre-closed (β − closed) if int(cl(int(A))) ⊆ A with $C_{int(cl(int(A)))(x)} \leq C_{A(x)}$, ∀x ∈ X.

(v) A pre-open if A ⊆ int(cl(A)) with $C_{A(x)} \leq C_{int(cl(A))(x)}$,∀x ∈ X.

(vi) A pre-closed if cl(int(A)) ⊆ A with $C_{cl(int(A))(x)} \leq C_{A(x)}$,∀x ∈ X.

(vii) An α-open if A ⊆ int(cl(int(A))) with $C_{A(x)} \leq C_{int(cl(int(A)))(x)}$, ∀x ∈ X.

(viii) An α-closed if cl(int(cl(A))) ⊆ A with $C_{cl(int(cl(A))(x)} \leq C_{A(x)}$,∀x ∈ X.

(ix) A regular-open if A = cl(int(A)) with $C_{A(x)} = C_{cl(int(A))(x)}$, ∀x ∈ X.

(x) A regular-closed if int(cl(A)) = A with $C_{int(cl(A))(x)} = C_{A(x)}$, k∀x ∈ X.

Definition 2.9. .[8] Let U, V be two non-empty sets, $[U]^w$, $[V]^r$ be two multiset spaces on U andV respectively. The ordered pair (M_1, M_2) is called a binary multiset (or bms) where $M_1 \in [U]^w$, $M_2 \in [V]^r$ are two multisets drawn from U and V respectively.

Note 2.10. .[8] We know that, the power set of a multiset M_1 (resp.M_2) is the support set of the power multiset of M_1 (resp.M_2), is symbolized by $P^*(M_1)$ (resp.$P^*(M_2)$). We can define $P^*(M_1) \times P^*(M_2) = \{(A_i, B_i): A_i \in P^*(M_1), B_i \in P^*(M_2)\}$. According this definition, the ordered pair (A, B) is called a bms from M_1 and M_2 where $A \subseteq M_1$ and $B \subseteq M_2$. That is, the bms (A, B) is an element in $P^*[M_1] \times P^*[M_2]$.

Definition 2.11 .[8] Let $M_1 \in [U]^w$, $M_2 \in [V]^r$ be two multisets drawn from U and V respectively. A binary multiset topology (briefly, bms-topology) from M_1 to M_2 is a binary multiset structure $\tau_b \subseteq P^*(M_1) \times P^*(M_2)$ that satisfies the following axioms:

(i) (φ, φ),(M_1, M_2) ∈ τ_b.
(ii) If (A_1, B_1),(A_2, B_2) ∈ τ_b, then (A_1 ∩ A_2),(B_1 ∩ B_2) ∈ τ_b.
(iii) If {(A_λ, B_λ) : λ ∈ J} ⊆ τ_b, then (∪A_λ, ∪B_λ) ∈ τ_b.
In this case, the structure (M_1, M_2, τ_b) is called bms-topological space (or bms-space).
Note that τ_b is an ordinary set whose elements are bms.
Definition 2.12. .[8] For a bms-space (M_1, M_2, τ_b), we have
(i) Each element in τ_b is called an open binary multiset (or open bms) and the complement of open bms is named a closed binary multiset (or closed bms).
(ii) A sub-bms (A,B) of a bms-space (M_1, M_2, τ_b) is said to be closed bms if the bms (A, B) ^{c} =(M_1 A, M_2 B) is an open bms.
Definition 2.13. .[8] Let (M_1, M_2, τ_b) be an bms topological space and A ⊆ M_1 , B ⊆ M_2. Then (A,B) is binary multiset closed in (M_1, M_2, τ_b) if (M_1 A, M_2 B) ∈ τ_b, the complement of closed bms $\tau^c{}_b$
Example 2.14. .[8] Let M_1 = {2/a, 1/b, 2/c}, M_2 = {1/d, 3/e, 2/f} be a two m-set. Consider, the bms topological space τ_b = {(φ, φ),(M_1, M_2),({2/a}, {3/e}),({1/b}, {2/f}),({2/c}, {2/b}, {1/a})} is an binary multiset topological space. Now ({2/a}, {3/e}),({1/b}, {2/f}),({2/c}, {2/b}, {1/a}) is an binary multiset open. Since ({1/b, 2/c}, {1/d, 2/f}),({2/a, 2/c}, {1/d, 3/e}),({2/a, 1/b}, {2/a, 1/b}, {1/a, 1/b, 2/c}) are bms closed.
Definition 2.15.[8] The ordered pair (A_1, B_1)*,(A_2, B_2)* is called bms closure of (A,B) is defined asthe intersection of all closed bms containing in (A,B) denoted by cl_b ({A, B}) is bms topological space (M_1, M_2, τ_b) where (A, B) ⊆ (M_1, M_2), cl_b ({A, B}) = ∩{(G, H) ⊆ (M_1, M_2) : (G, H) is a closed bms and (A,B) ⊆ (G,H)}.
3 Fundamental Properties of Open BMS and Closed BMS
In this section, we explore the concepts of bms semi-open, bms semi pre-open, bms pre-αopen, bms βopen, bms -regular open, bms γ-open, bms ω -open, bms b-open, bms b-open, and bms regular α-open, along with their respective theorems and examples.
Definition 3.1. Let (A,B) be a sub-bms of the bms-topological space (M1,M2, τ_b). The pair of (A,B) termed is bms semi-open if (A,B) ⊆ clb(intb({A,B})). Moreover, it is called bms semi-closed if intb(clb({A,B})) ⊆ (A,B).
Example 3.2. Consider M1 = {1/s, 1/r} and M2 = {2/r} as two multiset elements, with
τb = {(φ,φ)(M1,M2), ({1/sg), ({1/r}), ({1/sg, {1/r})}. Since, ⊆ c
τb = f(φ,φ), (M1,M2),({1/r}, {1/s} implies that (A,B) = ({1/r}). Therefore, intb({A,B}) = ({1/r}) and if clb({A,B}) = ({1/r}), then clb(intb({A,B})) ⊆ ({1/r}). Furthermore, intb(clb({A,B})) ⊆ ({1/r}) is also bms semi-closed.
Theorem 3.3. A sub-bms (A,B) in a (M1,M2, τb) is bms semi-open if and only if (A,B) = intb({A,B}).
Proof. Assume (A,B) that sub-bms of (M1,M2) in every open bms is know as bms semiopen, and the complement of a bms semi-open is bms semi-closed. A bms semi-open satisfies (A,B) ⊆ intb({A,B}). A closed bms is defined by (A,B) ⊆ clb({A,B}) ⊆ $\tau^c{}_b$. Since(A,B) ⊆ clb(intb({A,B})), it follows that (A,B) = intb({A,B}).
Theorem 3.4. A sub-bms (A,B) of the bms-topological space (M1,M2, τ_b) is classified as bms semi-closed if and only if (A,B) = clb({A,B}).
Proof. Let (A,B) be a sub-bms of (M1,M2). The complement of an open bms is closed bms , which means that a bms semi-closed satisfies (A,B) ⊆ clb({A,B}) ⊆ τb. Conversely, the complement of a closed bms is bms semi-open, given by (A,B) ⊆ intb({A,B}) ⊆ $\tau^c{}_b$. Thus, we have intb(clb({A,B})) ⊆ (A,B) ⊆τb.
Definition 3.5. Let (M1,M2, τ_b) be a bms-topological space with sub-bms (A,B). The pair (A,B) is termed bms semi pre-open (or bms -open) if (A,B) ⊆ clb(intb(clb({A,B}))). It is considered bms semi pre-closed if intb(clb(intb({A,B}))) ⊆ (A,B).
Example 3.6. Let M1 = {2=a, 1/b} and M2 = {1/a, 1/b} be two multisets, with τb = f{(φ,φ), (M1,M2), ({1/a}, {1/b}), ({1/a, 1/b}} and ⊆ τ^cb .
τ_b = {(φ,φ), (M1,M2), ({1/a, 1/b}, {1/a}),
({1/a}, {1/a, 1/b}}. Thus, (A,B) = ({1/a, 1/b}) ⊆ (M1,M2) if intb({A,B}) = ({1/a, 1/b}) and clb({A,B}) = ({1/a, 1/b}). Here, clb(intb(clb({A,B}))) = ({1/a, 1/b}). Therefore,

intb(clb(intb({A,B}))) = ({1/a, 1/b}).
Theorem 3.7. The bms semi pre-open if and only if (A,B) = bmsspint({A,B}) for the sub-bms (A,B) in the bms-topological space (M1,M2, τb).
Proof. Since (A,B) ⊆ (M1,M2), the complement of an open bms is classified as a closed bms. Thus, a bms semi pre-open satisfies (A,B) ⊆ intb({A,B}) ⊆ τ_b ,Additionally, (A,B) ⊆ clb({A,B}) ⊆ $\tau^c{}_b$ ..Given that (A,B) ⊆ clb(intb(clb({A,B}))), it follows that (A,B) is a bms semi pre-int (A,B).
Proposition 3.8. A bms semi pre-closed if and only if (A,B) = bmsspclb({A,B}).
Definition 3.9. A bms pre-open if (A,B) ⊆ intb(clb({A,B})). Consequently, clb(intb({A,B})) ⊆ (A,B) indicates that it is bms pre-closed.
Example 3.10. Consider M1 = {1/x} and M2 = {2=y}, Since τ_b = {(ϕ,ϕ), (M1,M2), ({1/x}, {1/y}), ({1/x, 2/y})} and
τ_b ={(φ,φ), (M1,M2), ({1/y})}. Thus, (A,B) = {1/y}
where intb({A,B}) = {1/y} and clb({A,B}) = {1/y}. Since intb(clb({A,B})) ⊆ {1/y} (A,B) is bms pre-open, and clb(intb({A,B})) ⊆ {1/y}
Theorem 3.11. A sub-bms (A,B) in a bms-topological space (M1,M2, τb) is pre-open if and only if (A,B) = bmsspintb({A,B}).
Proof. Since (A,B) ⊆ (M1,M2) is a bms-topological space, we have intb({A,B}) ⊆ τb and for a bms pre-closed, clb({A,B}) ⊆ $\tau^c{}_b$. Therefore, (A,B) ⊆ intb(clb({A,B})).
Theorem 3.12. The bms pre-closed if and only if (A,B) = bmspclb({A,B}) for the sub-bms (A,B) in a bms-topological space (M1,M2, τb).
Proof. Since (A,B) is a sub-bms of (M1,M2), we know that for a bms pre-closed, intb({A,B}) ⊆ τb. The complement of a bms pre-closed is a bms pre-open set, meaning clb({A,B}) ⊆ $\tau^c{}_{b,}$ so that clb(intb({A,B})) ⊆ (A,B).
Definition 3.13. A bms α-open if (A,B) ⊆ intb(clb(intb({A,B}))), while a bms is α-closed if clb(intb(clb({A,B}))) ⊆ (A,B).
Example 3.14. Let M1 = {3/a, 2/s} and M2 = {2/a} be given by τb = {(φ,φ), (M1,M2), ({2/s}, {2/a}), ({2/s, 2/a})} and
τ_b = {(φ,φ), (M1,M2), ({3/a}), ({1/a})}. Thus, (A,B) =
{1/a}, with intb({A,B}) = {1/a} and clb({A,B}) = {1/a}. Since
intb(clb(intb({A,B}))) ⊆ {1/a}, it is a bms α-open, and clb(intb(clb({A,B}))) ⊆ {1/a}.
Definition 3.15. A bms rα-open if (A,B) = intb(clb(intb({A,B}))), and it is rα-closed if clb(intb(clb({A,B}))) = (A,B).
Example 3.16. Consider M1 = {1/p, 1/r} and M2 = {1/p, 2=q, 1/r} be given by τb = {(φ,φ), (M1,M2), ({1/p}, {2/q}), ({1/p, 2/q}, {1/p, 1/r}), ({1/r}, {1/p})} and
$\tau^c{}_b$ = {(φ,φ),(M1,M2), ({1/r}, {1/p, 1/r}), ({1/r}), ({1/p}, {1/r, 2/q})}. Let (A,B) = {1/p, 1/r}. We have intb({A,B}) = {1/p, 1/r} and clb({A,B}) = {1/p, 1/r}. Since intb(clb(intb({A,B}))) = {1/p, 1/r}, it is rα-open, and clb(intb(clb({A,B}))) = {1/p, 1/r}.
Proposition 3.17. A sub-bms (A,B) in the bms-topological space (M1,M2, τb) is rα-open if and only if (A,B) = bms rα⊆ intb({A,B}).
Proposition 3.18. A sub-bms (A,B) in the bms-topological space (M1,M2, τb) is rα-closed if and only if (A,B) = bms rα⊆ clb({A,B}).
Definition 3.19. Let (A,B) be a sub-bms in (M1,M2, τb). It is bms b-open if (A,B) ⊆ clb(intb({A,B})) [intb(clb({A,B})), and it is bms b-closed if clb(intb({A,B})) [intb(clb ({A,B})) ⊆ (A,B).
Example 3.20. If M1 = {2/m} and M2 = {1/s}, since τb = {(φ,φ), (M1,M2), ({1/s}, {1/m}), ({1/s, 1/m})} and
$\tau^c{}_b$ = {(φ,φ), (M1,M2), ({1/m}), ({1/m})}. Let
(A,B) = {1/m} with intb({A,B}) = {1/m} and clb({A,B}) = {1/m}. Therefore, clb(intb ({A,B})) = {1/m} is bms b-open, and intb(clb({A,B})) = {1/m} indicates that ({1/m}) is bms b-closed.
Definition 3.21. A bms is ω-open if (A,B) ⊆ clb(intb({A,B})), it is ω-closed if intb(clb({A,B})) ⊆ (A,B).

Example 3.22. Let M1 = {2/p} and M2 = {2/q} be the τb = {(φ,φ), (M1,M2), ({1/p}, {1/q}), ({1/p, 1/q})} and τ^{c}_{b} = {(φ,φ), (M1,M2), ({1/p}, {1/q}), ({1/p, 1/q})}. Let (A,B) = {1/p} with intb({A,B}) = {1/p} and clb({A,B}) = {1/p}. Thus, clb(intb({A,B})) = {1/p} is ω-open, and intb(clb({A,B})) = {1/p} is ω-closed.

Definition 3.23. A bms γ-open if (A,B) ⊆ clb(intb({A,B})) [intb(clb({A,B})), while it is γ-closed if intb(clb({A,B})) \ clb(intb({A,B})) ⊆ (A,B).

Example 3.24. Consider M1 = {1/l, 1/n} and M2 = {1/n}, τb = {(φ,φ), (M1,M2), ({1/l}, {1/n}), ({1/l, 1/n})} and τ^{c}_{b} = {(φ,φ), (M1,M2), ({1/n})}. For (A,B) = {1/l}, we have intb({A,B}) = {1/l}, clb({A,B}) = {1/l}, and clb(intb({A,B})) [intb(clb({A,B})) = {1/l}. Moreover, intb(clb({A,B})) \ clb(intb({A,B})) = {1/l} shows it is γ-closed.

Definition 3.25. A sub-bms regular-open if (A,B) = clb(intb({A,B})), the complement of a regular-open bms is a regular-closed bms if intb(clb({A,B})) = (A,B).

Example 3.26. Let M1 = {1/s, 1/r} and M2 = {1/r, 1/s} be a multiset with the bms topology τb = f(φ,φ), (M1,M2), ({1/s}, {1/r}), ({1/s, 1/r})} and τ^{c}_{b} = {(φ,φ), (M1,M2), ({1/r}, {1/s}), ({1/s, 1/r})}.Let (A,B) = {1/r} with intb({A,B}) = {1/r} and clb({A,B}) = {1/r}. Therefore, clb(intb({A,B})) = {1/r} indicates it is regular-open, and intb(clb({A,B})) = {1/r}.

4 Comparison Based on Open and Closed Binary Multiset

We define the comparison of bms semi-open, bms semi-closed, bms pre-open, pre-closed, bms α-open, bms α-closed, regular-open, and regular-closed in this section.

Theorem 4.1. Every closed bms in (M1,M2) is bms α-closed.

Proof. Let (A,B) be a closed bms in (M1,M2). Let (S, T) be any bms α-open in (M1,M2) suchthat (A,B) ⊆ (S, T). Since (A,B) is a closed bms, we have clb({A,B}) = (A,B). Therefore,bms α clb({A,B}) ⊆ clb({A,B}) ⊆ (S, T). Hence, (A,B) is bms α-closed in (M1,M2).

Theorem 4.2. Every bms α-closed in (M1,M2) is bms rα-closed.

Proof. Let (A,B) be a bms α-closed in the bms-topological space (M1,M2), so that (U, V) bms rα-open in (M1,M2) such that (A,B) ⊆ (U, V). Since (A,B) is bms α-closed, we have bms α clb({A,B}) = (A,B) ⊆ U. Therefore, bms α clb({A,B}) ⊆ (U, V). Hence, (A,B) is bms rα-closed.

Theorem 4.3. Every open bms in (M1,M2) is bms α -open.

Proof. Let (A,B) be an open bms in (M1,M2). So that, (S, T) any bms α-closed in (M1,M2) such that (A,B) ⊆ (S, T). Since (A,B) is an open bms, we have clb({A,B}) = (A,B). Therefore, bms α intb({A,B}) ⊆ intb({A,B}) ⊆ (S, T). Hence, (A,B) is bms α -open in (M1,M2).

Theorem 4.4. Every bms α -open set in (M1,M2) is bms rα-open.

Proof. Let (A,B) be a bms α -open in (M1,M2). Let (U, V) be a bms rα-open in (M1,M2) such that (A,B) ⊆ (U, V). Since (A,B) is bms ⊆-open, we have bms ⊆ intb({A,B}) = (A,B) ⊆ (U, V). Therefore, bms α intb({A,B}) ⊆ (U, V). Hence, (A,B) is bms rα-open.

Theorem 4.5. The union of two bms γ -open is bms γ -open.

Proof. Let (A,B) and (C,D) be two bms γ-open in (M1,M2, τb). Let (G,H) be any closed bms in (M1,M2, τb) such that (A,B) [(C,D)]⊆τb. Then, (A,B) ⊆ G and (C,D) ⊆ (G,H). Since (A,B) and (C,D) are bms γ -open, we have bms γ intb({A,B}) ⊆ (G,H) and bms γ intb(C,D) ⊆ (G,H). Therefore, bms intb({A,B}) [bms intb(C,D) ⊆ (G,H), which implies bms γ intb((A,B) [(C,D)) ⊆ (G,H). Hence, (A,B) [(C,D) is bms γ -open.

Theorem 4.6. The intersection of two bms ω-closed is bms ω -closed.

Proof. Let (A,B) and (C,D) be two bms ω -closed in (M1,M2, τb). Let (G,H) be any open bms in (M1,M2, τb) such that (A,B) \ (C,D) ⊆ (G,H). Then, (A,B) ⊆ (G,H) and (C,D) ⊆ (G,H). Since (A,B) and (C,D) are bms ω -closed, we have bms ω clb({A,B}) ⊆ (G,H) and bms clb(C,D) ⊆ (G,H). Therefore, bms ω clb({A,B})\bms ω clb(C,D) ⊆ (G,H). Hence, (A,B) \ (C,D) is bms ω -closed.

Theorem 4.7. Every open bms in (M1,M2) is bms pre-open in (M1,M2).

Proof. Since (A,B) is an open bms (M1,M2, τb), let (G,H) be any bms pre-closed in (M1,M2) such that (A,B) ⊆ (G,H). Thus, intb(clb({A,B})) ⊆ intb({A,B}), which implies intb({A,B}) ⊆ intb({A,B}) ⊆ (G,H). Hence, intb(clb({A,B})) ⊆ (G,H).
Remark 4.8. The following diagram representing relationships in the bms-topological space is a reversible process, not an irreversible one.

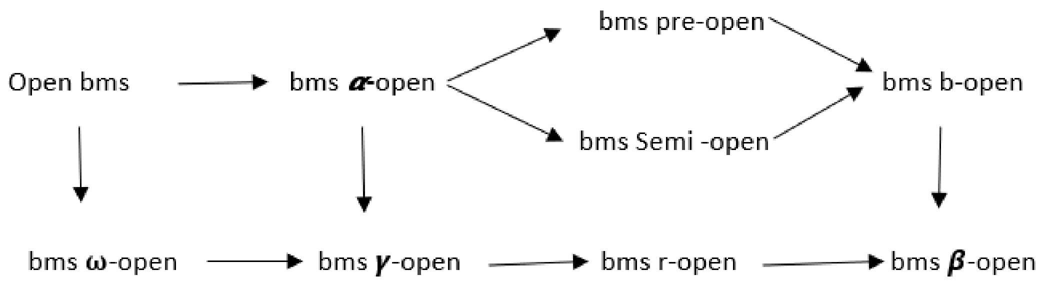

Figure 1

5 Conclusion

In this research, we present the concept of binary multiset topological spaces, providing a comprehensiveframework that encompasses various types of bms structures, including closed bms , open bms, and numerous other specialized classes such as interior bms, closure bms, sub-bms,bms semi-open, bms semi-pre-open, bms pre-open, bms-open, bms -open, bms -open, bms-open, bms b-open, and bms regular-open.

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